Coupled Transmission Lines

MAHESH KUMAR, STUDENT MEMBER, IEEE, AND B. N. DAS

Abstract—**An analysis of coupling between a stripline and a rectangular waveguide is presented. Coupling is through small apertures in the form of a circular hole or a narrow rectangular slot. A closed form expression for the normalized field distribution in the stripline, required for the purpose of analysis, is also derived. The theoretical results show good agreement with the experimental results.**

I. Introduction

A NUMBER OF investigations have been made on coupling between transmission lines [1]–[3]. In the present work, the problem of coupling between a rectangular waveguide and a stripline, coupled through an aperture in the form of a circular hole or a narrow rectangular slot, is investigated. The coefficient of coupling can be determined from the equivalent electric dipole moment, which is proportional to the electric field normal to the aperture plane, and the equivalent magnetic dipole moment, which is linearly related through a matrix relationship to the magnetic field tangential to the aperture plane. A closed form expression giving the normalized electric and magnetic field distributions in a stripline is derived and utilized to determine the coupling between a rectangular waveguide and a stripline. The variation of the coupling coefficient between a rectangular waveguide and a stripline coupled through a single rectangular slot or a circular hole is computed as a function of the aperture dimension. The comparison of theoretical results with experimental data is presented.

II. The Expression for Normalized Field Distribution in a TEM-Mode Stripline

Consider the symmetrical stripline of Fig. 1. The region between ground planes is assumed to be filled with a medium having the relative dielectric constant ε_y. The field distribution inside the stripline can be found from the complex potential function. This function can be obtained by applying two successive conformal transformations. It is of the form [4]

$$W' = \frac{sn^{-1}[(-W)^{1/2},k] - sn^{-1}(1/k,k)}{sn^{-1}(1,k) - sn^{-1}(1/k,k)} \tag{1}$$

where W in (1) is related to $Z = x + jy$ of Fig. 1 by the expression [4]

$$Z = -\frac{2a}{\pi}\ln\left[W^{1/2} + (W+1)^{1/2}\right] + ja \tag{2}$$

and

$$k = \left(\cosh\frac{\pi d}{4a}\right)^{-1}.$$

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The authors are with the UHF and Microwave Laboratory, Department of Electronics and Electrical Communication Engineering, Indian Institute of Technology, Kharagpur, India.

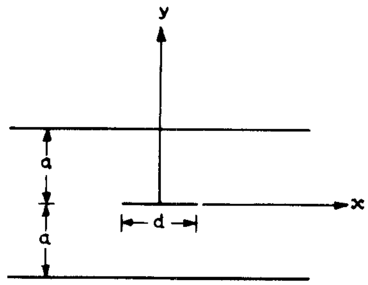

Fig. 1. Symmetrical strip transmission line (cross section).

From (2), it is found that

$$W = -\cosh^2\frac{\pi Z}{2a}. \tag{3}$$

The negative of the complex conjugate of the derivative of the complex potential function in (1) gives a closed form expression of the electric field distribution inside the stripline [5]. On taking the derivative of the complex potential which involves the derivative of an elliptic integral with complex argument [6], the electric field distribution function is obtained as

$$E = -\frac{\pi A}{2aK'}\left[\frac{1}{(1+k^2W)^{1/2}}\right]^* \tag{4}$$

where A is a constant, the asterisk denotes the complex conjugate, and K' is the complete elliptic integral of the first kind of modulus $k' = (1-k^2)^{1/2}$.

The electric field obtained from (4) can be written as

$$\boldsymbol{E} = \boldsymbol{a}_x E_x + \boldsymbol{a}_y E_y \tag{5}$$

while the magnetic field is given by [4]

$$\boldsymbol{H} = (\bar{I})\cdot\boldsymbol{E} \tag{6}$$

where $(\bar{I})$ is an identity dyadic given by

$$(\bar{I}) = \boldsymbol{a}_y\boldsymbol{a}_x - \boldsymbol{a}_x\boldsymbol{a}_y.$$

Following Harrington's method [7] the normalized electric field distribution in the transverse plane of the stripline can be found from the condition

$$\iint_s \boldsymbol{E}\times\boldsymbol{H}\cdot\boldsymbol{a}_z\, ds = 1. \tag{7}$$

From (4) to (7), the constant A for normalization is found to be

$$A = \frac{2K'}{\pi\sqrt{F(k)}} \tag{8}$$

where

$$F(k) = \frac{1}{a^2}\int_{-\infty}^{\infty}\int_{-a}^{a}\frac{dx\,dy}{|1+k^2W|}. \tag{9}$$

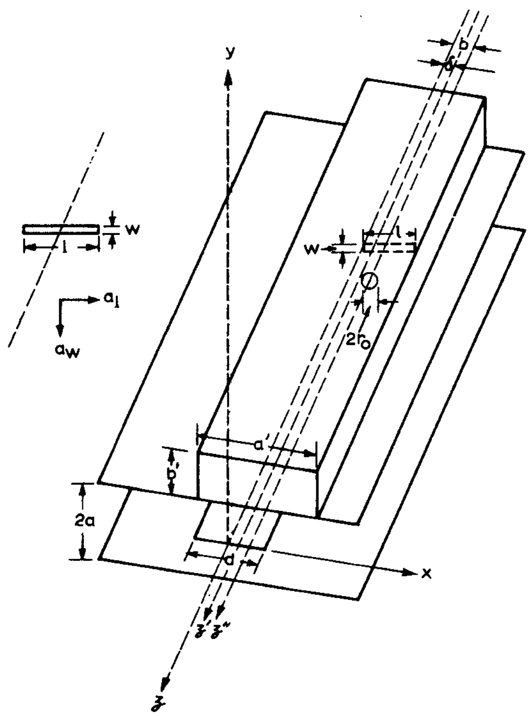

Fig. 2. Schematic of a stripline-to-waveguide coupler. z: Longitudinal axis of the stripline. z': Longitudinal axis of the rectangular waveguide (parallel to z). z'': Axis of symmetry of the aperture (parallel to z and z').

From (4) to (9), the normalized electric and magnetic fields for a propagating TEM-mode wave are given in complex form by

$$E = -\frac{1}{a\sqrt{F(k)}}\left[\frac{1}{(1+k^2W)^{1/2}}\right]^* e^{-j\beta z} \tag{10a}$$

$$H = \frac{Y_0}{a\sqrt{F(k)}}\left[\frac{1}{(1+k^2W)^{1/2}}\right]^* e^{-j\beta z} \tag{10b}$$

where β is the propagation constant and Y_0 is the characteristic admittance of the stripline.

III. A Small Circular Hole in the Common Wall Between Two Transmission Lines

Consider a rectangular waveguide coupled to a stripline through a circular hole of radius r_0 ($r_0 \ll \lambda$) in the common wall (Fig. 2). Assume that the relative dielectric constants for the stripline and the rectangular waveguide are $\varepsilon_{1\gamma}$ and $\varepsilon_{2\gamma}$, respectively, and that the TE_{10} wave of unit amplitude is incident in the rectangular waveguide.

Let the field radiated by the electric dipole be

$$\boldsymbol{E} = \begin{cases} A_1\boldsymbol{E}^+, & z > 0 \\ A_2\boldsymbol{E}^-, & z < 0 \end{cases}$$

$$\boldsymbol{H} = \begin{cases} A_1\boldsymbol{H}^+, & z > 0 \\ A_2\boldsymbol{H}^-, & z < 0 \end{cases}$$

whereas that radiated by the magnetic dipole is

$$\boldsymbol{E} = \begin{cases} A_3\boldsymbol{E}^+, & z > 0 \\ A_4\boldsymbol{E}^-, & z < 0 \end{cases}$$

$$\boldsymbol{H} = \begin{cases} A_3\boldsymbol{H}^+, & z > 0 \\ A_4\boldsymbol{H}^-, & z < 0 \end{cases}$$

where $\boldsymbol{E}^+$ and $\boldsymbol{H}^+$ are the normalized electric and magnetic fields in the line due to a source placed at $z = -\infty$, and $\boldsymbol{E}^-$ and $\boldsymbol{H}^-$ are the normalized electric and magnetic fields in the line due to a source placed at $z = \infty$. For radiation in the coupled stripline, $\boldsymbol{E}^+$ and $\boldsymbol{H}^+$ are given by (10a) and (10b), respectively, while $\boldsymbol{E}^-$ and $\boldsymbol{H}^-$ are obtained from the same expressions on replacing z by $-z$. For radiation in the coupled rectangular waveguide in the dominant mode, $\boldsymbol{E}^+$, $\boldsymbol{H}^+$, $\boldsymbol{E}^-$, and $\boldsymbol{H}^-$ are given by the following expressions:

$$\boldsymbol{E}^+ = \sqrt{\frac{2}{a'b'}}\,\boldsymbol{a}_y \cos\frac{\pi(x-\delta)}{a'}\, e^{-j\beta z}, \qquad z > 0 \tag{11a}$$

$$\boldsymbol{H}^+ = -\sqrt{\frac{2}{a'b'}}\, Y_\omega \left[\boldsymbol{a}_x \cos\frac{\pi(x-\delta)}{a'} + \frac{j\pi}{\beta a'}\boldsymbol{a}_z \sin\frac{\pi(x-\delta)}{a'}\right] e^{-j\beta z}, \qquad z > 0 \tag{11b}$$

$$\boldsymbol{E}^- = \sqrt{\frac{2}{a'b'}}\,\boldsymbol{a}_y \cos\frac{\pi(x-\delta)}{a'}\, e^{j\beta z}, \qquad z < 0 \tag{11c}$$

$$\boldsymbol{H}^- = \sqrt{\frac{2}{a'b'}}\, Y_\omega \left[\boldsymbol{a}_x \cos\frac{\pi(x-\delta)}{a'} - \frac{j\pi}{\beta a'}\boldsymbol{a}_z \sin\frac{\pi(x-\delta)}{a'}\right] e^{j\beta z}, \qquad z < 0 \tag{11d}$$

where Y_ω is the wave admittance of the dominant TE_{10} mode, the longitudinal axis of the stripline is given by $x = 0$, and $x = \delta$ is the longitudinal axis of the rectangular waveguide.

The expressions for A_1, A_2, A_3, and A_4 have been derived by Collin [9] using the Lorentz reciprocity theorem. It is found that

$$A_1 = A_2 = -\frac{j\omega}{P_n}\boldsymbol{E}^- \cdot \boldsymbol{P} \tag{12a}$$

$$= -\frac{j\omega}{P_n}\boldsymbol{E}^+ \cdot \boldsymbol{P} \tag{12b}$$

$$A_3 = \frac{j\omega\mu}{P_n}\boldsymbol{H}^- \cdot \boldsymbol{M} \tag{12c}$$

$$A_4 = \frac{j\omega\mu}{P_n}\boldsymbol{H}^+ \cdot \boldsymbol{M}. \tag{12d}$$

P_n appearing in expressions (12) is found to be of the form

$$P_n = 2\iint \boldsymbol{E}^+ \times \boldsymbol{H}^+ \cdot \boldsymbol{a}_z\, ds = 2\iint \boldsymbol{E}^- \times \boldsymbol{H}^- \cdot \boldsymbol{a}_z\, ds \tag{13}$$

where the integration extends over the cross section of the coupled line. The equivalent electric and magnetic dipole moments are given by [8]

$$P = \frac{\varepsilon_{1y}\varepsilon_{2y}\varepsilon_0}{(\varepsilon_{1y} + \varepsilon_{2y})} (\bar{\tau}) \cdot E \tag{14a}$$

$$M = (\bar{\rho}) \cdot H_t \tag{14b}$$

where the dyadics $(\bar{\tau})$ and $(\bar{\rho})$ are given by [4], [8]

$$(\bar{\tau}) = \frac{4}{3} r_0^3 a_n a_n$$

$$(\bar{\rho}) = \frac{4}{3} r_0^3 (\bar{I}_t).$$

a_n is the unit vector normal to the aperture plane, and $(\bar{I}_t)$ is the identity dyadic in the boundary plane

$$(\bar{I}_t) = a_x a_x + a_y a_y.$$

For a circular hole located at $x = b$, $y = a$, and $z = 0$, P and M are found to be

$$P = \frac{4}{3}\sqrt{\frac{2}{a'b'}} \frac{\varepsilon_{1y}\varepsilon_{2y}\varepsilon_0 r_0^3 a_y \cos \pi(b - \delta)/a'}{(\varepsilon_{1y} + \varepsilon_{2y})} \tag{15a}$$

$$M = -\frac{4}{3}\sqrt{\frac{2}{a'b'}} r_0^3 a_x Y_\omega \cos \pi(b - \delta)/a'. \tag{15b}$$

The total field radiated into the coupled line is given by

$$E = \begin{cases} (A_1 + A_3)E^+, & z > 0 \quad (16a) \\ (A_2 + A_4)E^-, & z < 0 \quad (16b) \end{cases}$$

and

$$H = \begin{cases} (A_1 + A_3)H^+, & z > 0 \quad (16c) \\ (A_2 + A_4)H^-, & z < 0. \quad (16d) \end{cases}$$

The coupling is defined as [9]

$$C = -20 \log |A_1 + A_3|. \tag{17}$$

By using (10)–(17), the expression for C in the case of a hole located at $x = b$ is obtained as

$$C = -20 \log \left[\frac{2}{3}\sqrt{\frac{2}{a'b'}} \frac{\omega r_0^3}{a\sqrt{F(k)}} \cdot \left\{\frac{\varepsilon_{1y}\varepsilon_{2y}\varepsilon_0 \cos \pi(b - \delta)/a'}{(\varepsilon_{1y} + \varepsilon_{2y})Y_0 \left(1 + k^2 \sinh^2 \frac{\pi b}{2a}\right)^{1/2}} + \frac{\mu_0 Y_\omega \cos \pi(b - \delta)/a'}{\left(1 + k^2 \sinh^2 \frac{\pi b}{2a}\right)^{1/2}}\right\}\right]. \tag{18}$$

For the purpose of computation, it is assumed that the axes of the two transmission lines are parallel and are contained in the plane perpendicular to the common wall between the two transmission lines (i.e., $\delta = 0$). Calculations and experimental measurements are made for a circular hole with center located at $x = 0$, in the ground plane of a stripline of characteristic impedance 50 Ω, ground plane spacing $\frac{1}{8}$ in, and $\varepsilon_y = 2.56$, and in the broad wall of a rectangular waveguide having the dimensions $a' = 0.9$ in and $b' = 0.4$ in. The correction factor for

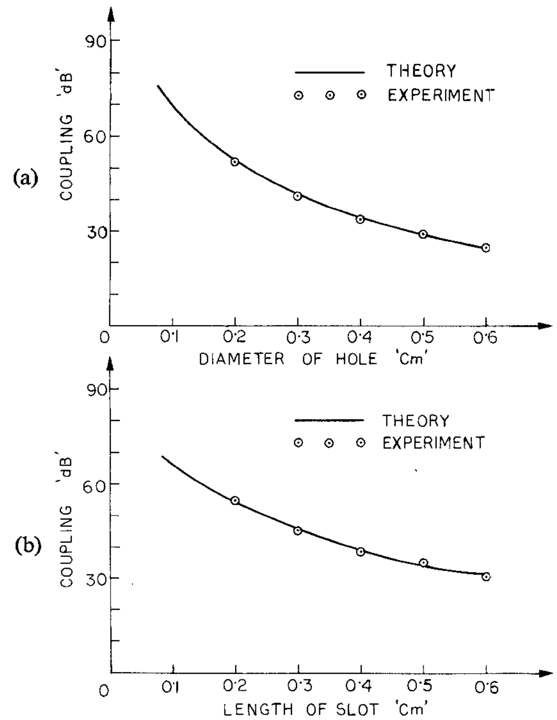

Fig. 3. (a) Variation of coupling C with the diameter of circular hole. (b) Variation of coupling C with the length of slot (slot width $w = 0.1$ cm).

apertures of diameter in the range of 0.1–0.6 cm is calculated by using the formula suggested in the literature [10], [11]. For this range of hole diameter, the coupling correction is below 0.4 dB. Since the coupler was built by notching out the waveguide wall and cutting the hole in the ground plane of the stripline (thickness 30 μm), the effect of wall thickness can be neglected. The coupling between a stripline and a rectangular waveguide is calculated for frequency of 9375 MHz from (18). In the experimental device fabricated, the effects of the higher order modes in the stripline are suppressed [12], [13] by placing metallic posts, of diameter 0.15 cm and longitudinal spacing 0.9 cm, on either side of the strip. The posts are laterally displaced 0.4 cm from the center of the strip. The experimental and theoretical results corrected for large aperture effects are presented in Fig. 3(a).

IV. A Narrow Rectangular Slot in the Common Wall Between Two Transmission Lines

When the two transmission lines are coupled through a rectangular slot as shown in Fig. 2, the $(\bar{\tau})$ and $(\bar{\rho})$ appearing in expressions (14) are given by [4], [8]

$$(\bar{\tau}) = \alpha_e a_n a_n \tag{19a}$$

$$(\bar{\rho}) = \alpha_m^l a_l a_l + \alpha_m^t a_w a_w. \tag{19b}$$

a_l and a_w appearing in (19) are the unit vectors along the length and width of the slot, respectively, and α_m^l and α_m^t are the longitudinal and transverse components of the magnetic polarizability. α_m^l can be obtained empirically from the data obtained by Cohn [14] from electrolytic tank measurements. An approximate expression for α_m^l

fitting the empirical data for the slot lengths up to $\lambda/6$ is given by [13], [14]

$$\alpha_m{}^l = 0.233wl^2 + 0.044l^3. \tag{20}$$

$\alpha_m{}^t$, however, can be obtained from Bathe's formula [3] as

$$\alpha_m{}^t = \frac{\pi w^2 l}{16}. \tag{21}$$

The electric polarizability α_e is given by [4]

$$\alpha_e = \frac{\pi w^2 l}{12}. \tag{22}$$

For a narrow rectangular slot, $\alpha_m{}^t$ and α_e are small compared to $\alpha_m{}^l$.

For a transverse slot, $\boldsymbol{a}_l = \boldsymbol{a}_x$ and $\boldsymbol{a}_w = \boldsymbol{a}_z$. Using expressions (11), (14), and (19)–(22), the electric and magnetic dipole moments are obtained as

$$\boldsymbol{P} = \frac{\varepsilon_{1y}\varepsilon_{2y}\varepsilon_0 \pi l w^2}{12(\varepsilon_{1y} + \varepsilon_{2y})} \sqrt{\frac{2}{a'b'}}\, \boldsymbol{a}_y \cos \frac{\pi(b-\delta)}{a'} \tag{23a}$$

$$\boldsymbol{M} = -(0.233wl^2 + 0.044l^3) \sqrt{\frac{2}{a'b'}}\, Y_\omega \boldsymbol{a}_x \cos \frac{\pi(b-\delta)}{a'}. \tag{23b}$$

From (10), (11)–(13), (16), (17), and (19)–(23), the expression for coupling between a rectangular waveguide and a strip line through a transverse rectangular slot located at $x = b$, $y = a$, and $z = 0$ is obtained as

$$C = -20 \log \left[\frac{\omega}{2a\sqrt{F(k)}} \sqrt{\frac{2}{a'b'}} \left\{ \frac{\varepsilon_{1y}\varepsilon_{2y}\varepsilon_0 \pi l w^2}{12 Y_0(\varepsilon_{1y} + \varepsilon_{2y})} + (0.233wl^2 + 0.044l^3) Y_\omega \mu_0 \right\} \cdot \frac{\cos \pi(b-\delta)/a'}{\left(1 + k^2 \sinh^2 \frac{\pi b}{2a}\right)^{1/2}} \right]. \tag{24}$$

Calculations and experimental measurements are made for the same configuration as described in Section III, with the exception that the circular aperture in the common wall is replaced by a transverse rectangular slot. The coupling between a stripline and a rectangular waveguide is calculated from (24) for slot width of 0.1 cm, as a function of the length of the slot. In the experimental device fabricated the effects of higher order modes in the stripline are suppressed [12], [13] by using metallic posts having the same dimension and spacing as those described in Section III. The theoretical results and experimental data are presented in Fig. 3(b).

V. Conclusions

The analysis for the coupling between a stripline and a rectangular waveguide is presented with the assumption that only the TEM mode exists in the stripline. A discontinuity in the stripline causes the excitation of higher order modes (mainly waveguide modes). When these higher order modes are suppressed by placing metallic posts with such a spacing that the rectangular waveguide modes excited by the aperture are below cutoff [12], [13], the experimental data are in close agreement with the theoretical results. In the absence of such posts, the deviation between experimental data and theoretical results is in the range of 2–3 dB.

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